MODULUS OF REAL NUMBERS

Solve the following for real x:

1. $|x| = 5$
2. $|x - 2| = 9$
3. $x + |x| = 0$
4. $|2x - 5| = 7$
5. $\left|\frac{3x+4}{3}\right| = 7$
6. $|x - 3| = -1$
7. $|x^2 - 5x + 2| = 4$
8. $|x + 2| = |x - 2|$
9. $|x - 1| = |2x - 1|$
10. $|x^2 + x - 1| = 2x - 1$
11. $|2x + 3| - |2x - 5| = 6$
12. Find the number of solutions of $|x| + |x - 1| = 5$
13. $|x + 2| + |x - 3| + 2 = 0$
14. $|x| < \sqrt{2}$
15. $|x - 3| < 5$
16. $1 \le |x - 1| \le 3$
17. $|x| > -3$
18. Solve simultaneously: $\begin{cases} |x| \ge 1, \\ |x-1| < 3 \end{cases}$
19. $|x^2 - 3x + 1| < 1$
20. Solve simultaneously: $\begin{cases} |x^2 - 4x| < 5, \\ |x+1| < 3 \end{cases}$
21. $\frac{2}{|x-4|} > 1$
22. $\left|\frac{2x-1}{x-1}\right| > 2$
23. $||x - 2| - 1| \ge 3$
24. $\frac{|x-2|}{x-2} > 0$
25. $|x - 2| \le |x + 4|$
26. $|x - 1| + |x - 2| \ge 4$
27. $|x| < \frac{2}{x}$
28. $x^2 - |x| - 2 \ge 0$
29. $\frac{|x-1|}{x+2} < 1$
30. $\frac{|x+3|+x}{x+2} > 1$
31. $\frac{1}{2-|x|} \ge 1$
32. $\frac{|x|-1}{|x|-2} \ge 0$
33. $|x^2 + x| > |x^2 + 5x - 8|$
34. $|-x^2 + 4x - 7| - x^2 + 4x - 7 = 0$
35. $\frac{x^2 - 7|x| + 10}{x^2 - 6x + 9} < 0$
36. $\left|\frac{x^2 - 3x - 1}{x^2 + x + 1}\right| < 3$
37. $\left|\frac{x^2 - 5x + 4}{x^2 - 4}\right| \le 1$
38. $|x| + \left|\frac{x}{x-1}\right| = \frac{x^2}{|x-1|}$
39. $|x^2 + 6x + 6| = |x^2 + 4x + 9| + |2x - 3|$
40. $|x^2 + 3x| > 2 - x^2$

ANSWER KEY

1. {–5, 5}
2. {–7, 11}
3. (–∞, 0]
4. {–1, 6}
5. (17/3, –25/3)
6. ϕ
7. $\left(\frac{5 \pm \sqrt{33}}{2},\ 2,\ 3\right)$
8. 0
9. $\left\{0, \frac{2}{3}\right\}$
10. $\left\{1, \frac{\sqrt{17}-3}{2}\right\}$
11. 2
12. 2
13. ϕ
14. $\left(-\sqrt{2},\ \sqrt{2}\right)$
15. (–2, 8)
16. [–2, 0] ∪ [2, 4]
17. $x \in R$
18. (-2, -1] ∪ [1, 4)
19. (0, 1) ∪ (2, 3)
20. (-1, 2)
21. (2, 4) ∪ (4, 6)
22. (3/4, 1) ∪ (1, + ∞).
    Hint. The given inequality is equivalent to the collection of two inequalities $\frac{2x-1}{x-1} > 2$ and $\frac{2x-1}{x-1} < -2$.
23. (–∞, –2] ∪ [6, ∞)
24. (2, + ∞)
25. [–1, ∞)
26. $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{7}{2}, \infty\right)$
27. $(0, \sqrt{2})$
28. (–∞, –2] ∪ [2, ∞)
29. (-∞, -2) ∪ (1/2, + ∞).
30. (–5, –2) ∪ (–1, ∞)
31. (–2, –1] ∪ [1, 2)
32. (–∞, –2) ∪ [–1, 1] ∪ (2, ∞)
33. (–∞, –4) ∪ (1, 2)
34. R
35. $x \in (-5, -2) \cup (2, 3) \cup (3, 5)$
36. (-∞, -2) ∪ (-1, + ∞).
    Hint. Since $x^2 + x + 1 > 0$ for all $x \in R$, the given inequality is equivalent to the inequality $|x^2 - 3x - 1| < 3(x^2 + x + 1)$, or to $-3(x^2 + x + 1) < x^2 - 3x - 1 < 3(x^2 + x + 1)$, or to $-3(x^2 + x + 1) < x^2 - 3x - 1$, $x^2 - 3x - 1 < 3(x^2 + x + 1)$.
37. [0, 8/5] ∪ [5/2, + ∞).
38. {0} ∪ (1, ∞)
39. $\left[\frac{3}{2}, \infty\right)$
40. $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$